ChE 330 – Fluid Mechanics and Transport Phenomena

Fall 2021

Midterm exam: **40 points**

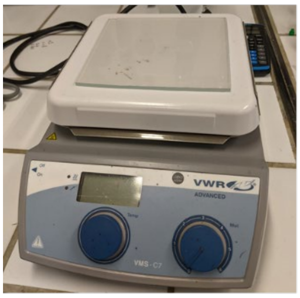
NAME, First Name________________________________

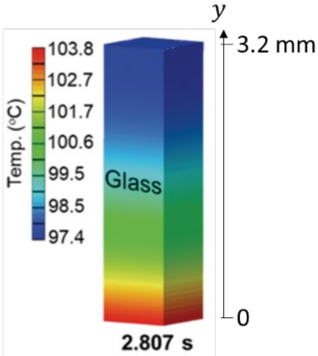
Signature_______________________________________

INSTRUCTIONS: You may use all of the course materials, module notes and exercise corrections for reference. A calculator may be used, but the **internet is not allowed**. Use additional sheets of paper to write your answers. Please show all of your derivations and calculations, but there is no need to re-derive equations already given in the course notes (just mention where the equation comes from). **Write your name on all sheets of paper used and staple them to this sheet when finished**. Write your name and signature above.

1. Heating a glass substrate (6 points)

Glass slab on a hotplate



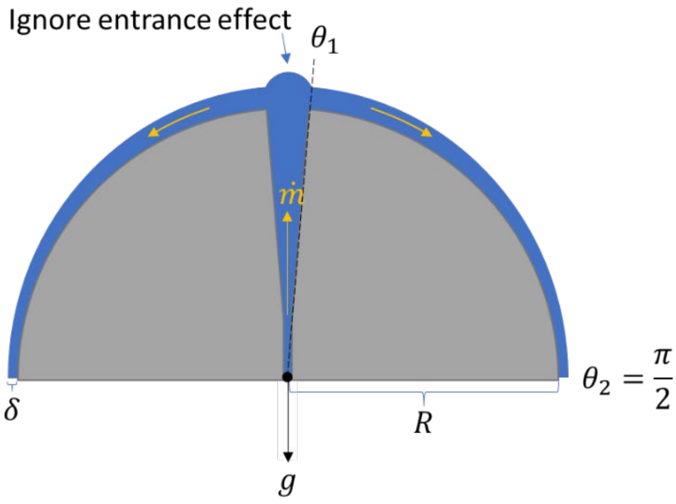


In a recent published paper on the perovskite solar cell [Li, N. *et al. Science* **373**, 561–567 (2021)], the authors were concerned about the heating of the perovskite thin film, which is coated on a large rectangular glass slab (3.2 mm in thickness) as a substrate. The authors performed a simulation to model the temperature of the glass, which was initially at 25 °C. At a time t = 2.807 seconds after the glass was placed on a hot plate with a temperature of 103.8 °C, they found the top surface of the glass to be at a temperature of 97.4°C. Can you estimate the thermal diffusivity of the glass?

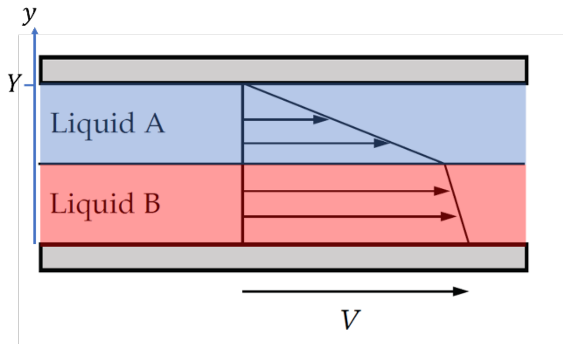
2. Mushroom fountain (11 points)

On a walk through a park you see this nice mushroom-shaped fountain where a steady-state flow rate of water, $\dot{m}$, flows up a central cone-shaped tube and then down the hemispherical surface of the fountain as indicated in the diagram. Assume that the flow is only driven by gravity (no pressure driven flow). Further assume that only steady-state Stokes flow occurs (not the case in the picture!) of a Newtonian fluid (constant ρ, μ) and disregard any entrance effects.

Can you propose a set of differential equations and boundary conditions that could be solved to find the fluid velocity as function of position as it flows over the hemispherical surface (i.e. from θ_1 to θ_2), and ultimately define the thickness of the film, δ? Do not solve these equations, but state any further assumptions that you make, and describe how to solve for δ. (Hints: assume a constant value for the velocity at θ_1, and use an integral mass balance at θ_2).



3. **Two phase flow (6 points)**
Two immiscible liquids A and B are flowing in steady laminar shear flow between two solid plates as illustrated in the figure. The bottom plate is moving to the right at a constant velocity V and the top plate is stationary. The velocity profile is indicated in the figure.



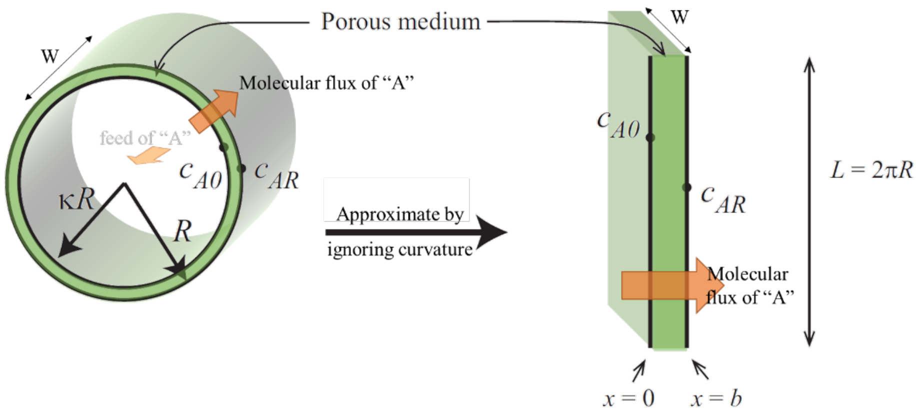
a) Which fluid has the larger viscosity? Provide an explanation for your answer with a mathematical statement, if possible.
b) Sketch the shear stress, τ_{yx} as a function of y.
c) Sketch the velocity profile for the two fluids if instead the top plate was moving at a constant velocity V and the bottom plate is stationary.

4. **Constant flux boundary condition (6 points)**
A special case of the semi-infinite slab solution for transient heat transfer can be defined when the boundary condition at the surface ($y = 0$) is a constant heat flux in the y direction (not a constant temperature as we saw in the course). Your colleague solves for this case and arrives at the following expression for the transient temperature profile :

$$T(y,t) = \frac{q_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(\frac{y^2}{4\alpha t}\right) - y\,\mathrm{erfc}\left(\frac{y}{\sqrt{4\alpha t}}\right)\right] + T_0$$

Where q_s is the constant heat flux at the surface, k is the thermal conductivity of the solid, T_0 is the initial temperature of the slab and the other symbols have their usual meanings. Do you think your colleague has arrived at the correct solution ? Justify your answer.

5. **Pharmaceutical production (11 points)**
In the production of a pharmaceutical chemical, an impurity chemical "A" is degraded in a thin, porous, cylindrical catalytic reactor (inner radius κR, outer radius R) as illustrated in the figure below. A dilute solution of reactant A is fed into the inner portion of the reactor ($r \leq$ κR). Species A then diffuses through the porous medium (where the effective diffusivity $\mathcal{D}_A$ is constant). The reactant also undergoes a first-order reaction in the porous medium:



$$\mathrm{A} \longrightarrow \text{products}$$

where the rate of consumption of the reactant A (R_A [=] moles A/volume·time) is expressed as $R_A = -k_1 c_A$ where k_1 is constant (and $k_1 > 0$ so that $R_A < 0$). We may assume that c_A only depends only on r, and as species A diffuses through and reacts with the porous medium, its concentration is reduced from c_{A0} at $r =$ κR to c_{AR} at $r =$ R. Before solving this problem, we first take advantage of the fact that the shell is thin, and therefore the cylindrical shell may be approximated as a thin slab of thickness $b = R - \kappa R$, and length $L \approx 2\pi R$, as illustrated on the right side of the figure above (the width into the page W is the same in both parts of the figure). The concentration of A now only depends on x ($\equiv r - \kappa R$).

(a) Solve for the steady-state concentration profile $c_A(x)$
(b) Write an expression for the total molecular flux (moles/time) of the reactant A that exits the porous medium at $x = b$.